ALGEBRA

Příklad. (Jarda) Find real numbers x, y and z for which following holds.

$$(x-y-3)^2+(y-z)^2+(x-z)^2=3$$

Příklad. (Kenny) Consider those functions $f:\mathbb{N}\mapsto\mathbb{N}$ which satisfy the condition

$$f(m+n)\geq f(m)+f((n))-1$$

for all $n\in\mathbb{N}$. Find all possible values of $f(2007)$.

GEOMETRIE

Příklad. (Jarda) A convex polyhedron is bounded by quadrilateral faces, its surface is A, and the sum of the squeares of its edges is Q. Prove that $Q\geq A$.

Příklad. (Jarda) M is the point in the interior of a given circle. The vertex of a right angle is M and its arms intersects the crcle at the points A and B. What is the locus of the midpoint of the line segment AB as the right angle is rotated about the point M?

Příklad. (Kenny) A rectangle D is partitioned in several (≥ 2) rectangles with sides parallel to those of D. Give that any line parallel to one of the sides of D, and having common points with the interior of D, also has common interior points with the interior of at least one rectangle of the partition; prove that there is at least one rectanfle of the partition having no common points with D's boundary.

KOMBINATORIKA

Příklad. (Jarda) Write down all integers from 1 to 10^{n-1}, and let A denote the number of difits hence writen down. Now write down all the numbers from 1 to 10^n, and let B denote the number of zeros written down this time. Prove that $A=B$.

Příklad. (Jarda) Nechť X je množina o n prvcích. Pro každou uspořádanou dvojici A, B podmnožin X označme $T_{A,B}$ počet prvků v průniku A a B. Spočtěte

$$\sum_{A,B} T_{A,B}$$

Příklad. (Kenny) Let $n>1$ be an integer. Find all sequences $a_1,a_2,\ldots,a_{n^2+n}$ satysfying the following conditions:

(a) $a_i\in 0,1$ for all $1\leq i\leq n^2+n$

(b) $a_{i+1}+a_{i+2}+\ldots+a_{i+n}<a_{i+n+1}+a_{i+n+2}+\ldots+a_{i+2n}$ for all $0\leq i\leq n^2+n$.

TEORIE CISEL

Příklad. (Jarda) The difference of two prime numbers is 100. If we concatenate them, we get another prime number. Find those numbers.

Příklad. (Jarda) Find all integers a such that $\frac{a^{2000}-1}{a-1}$ is a perfect square.

Příklad. (Kenny) Find all pairs (a,b) of natural numbers satysfying 7^a-3^b divides a^4+b^2