Communication

4TH AUTUMN SERIES — DATE DUE: 9TH JANUARY 2023

Pozor, u této série přijímáme pouze řešení napsaná anglicky!

PROBLEM 1. (3 POINTS)

A group of 26 pupils went to Matfyz. Every pupil is either always telling the truth or always lying. On their way there they went in pairs and everyone said "My partner is a liar." Could they have rearranged themselves in pairs, so that everyone said "My partner is always telling the truth"?

PROBLEM 2. (3 POINTS)

Petr, Denisa and Markéta are playing a game. First, Denisa tells Petr seven pairwise distinct positive integers whose sum is 100. Petr then tells Markéta the fourth highest of them and Markéta has to guess all seven numbers. The girls win if Markéta guesses all numbers correctly, but they cannot discuss their strategy beforehand. Which numbers should Denisa choose in order to win?

PROBLEM 3. (3 POINTS)

There are n piglets and each of them created one unique meme. One piglet can send a message to another containing all memes he currently knows. How many messages must be sent so that every piglet knows every meme?

PROBLEM 4. (5 POINTS)

Twenty teams competed in a tournament. On the first day, each team played (exactly) one game against one of the other teams. On the second day, every team played one game again, but this time against a different opponent. Prove that on the third day of the tournament, we can still choose ten teams such that no two of them played together before.

PROBLEM 5. (5 POINTS)

Distances between pairs of cities in PraSestan are unique.[1] Out of each city, an airplane departs to its nearest neighbor. Find the maximum number of airplanes which can end up in the same city.

PROBLEM 6. (5 POINTS)

An odd number of teams entered a two-day tournament. On the first day, every team faced every other team. The next day, they faced every other team again. Results of the tournament satisfy the following condition: each team lost as many times as it won and there were no draws. Prove that some half of the matches can be ignored so that the condition will still hold.

PROBLEM 7. (5 POINTS)

On Christmas Eve, n piglets exchanged some Christmas GIFs. We know that for each pair of piglets, at least one of them sent the other a Christmas GIF. We also know that every piglet received a Christmas GIF from exactly one quarter of the recipients of his Christmas GIFs. Determine all possible values of n.

[1] PraSestan can be represented as a plane where cities are points.

Problem 8. (5 points)

In the kingdom of Esarpia, there live $n \geq 3$ peasants. Some pairs of these peasants are friends[2]. We say that four peasants a, b, c and d form a *friendly square*, if the four pairs $\{a, b\}$, $\{b, c\}$, $\{c, d\}$ and $\{d, a\}$ are friends, but the pairs $\{a, c\}$ and $\{b, d\}$ are not.

King Esarp enacted a decree that a several day long festival is to be held to celebrate the birthday of his most trusted Archmage. The decree stipulates the following rules:

- On each day of the festival, a party must be held. Once no more parties can be held, the festival ends.
- A group of peasants may hold a party, only if they are all friends with each other and no peasant that does not participate is friends with all the participants.
- No two parties can have precisely the same set of participants.

Prove that if there are no friendly squares in Esarpia, then the festival can last at most $\frac{n(n-1)}{2}$ days.

[2]Friendship is symmetric.